Foreign Exchange Forward Rate Contracts

To assist in broadening the understanding of foreign exchange markets, WATC has provided the following example to demonstrate how the forward exchange rate is determined in a foreign exchange forward contract.

Overview

Foreign exchange forward contracts are transactions in which counterparties agree to exchange a specified amount of different currencies at some specified future date. The example demonstrates how a forward rate is derived from the interest rate differentials and hence, eliminates any opportunity for a risk-free profit. If the forward rate differs from that implied by the interest rate differential, an arbitrage opportunity arises. Arbitrage allows locking in a riskless profit by simultaneously entering into two or more transactions.

Assumptions

Let us assume the following:

- Spot rate AUD/USD 0.9550
- One year Australian interest rate 8.30% effective annual rate
- One year United States interest rate 3.20% effective annual rate.

Given these data, the one-year AUD/USD forward rate is 0.9100. Any forward rate, other than 0.9100, would allow for arbitrage and therefore the opportunity for a risk-free profit.

Arbitrage Example

Suppose the one year AUD/USD forward rate is less than 0.9100, say 0.9000. A portfolio that will yield a risk-free profit can be constructed as follows:

Time	Description	Cash Flows	
0	Borrow AUD 1M at 8.30%	AUD	0
	Convert to USD 955,000 and invest at 3.20%	USD	0
	Enter into a 1-year forward contract to buy AUD 1,083,000 at 0.9000 and sell USD 974,700		
	INITIAL OUTLAY	USD	0
1 year	Repay AUD loan	AUD	-1,083,000
	USD investment matures	USD	+985,560
	Receive AUD from forward contract	AUD	+1,083,000
	Outlay USD from forward contract	USD	-974,700
	INCOME	USD	+ 10,860

Therefore a profit of USD 10,860 can be achieved from a zero dollar outlay when the forward rate is 0.9000. Similarly, a profit opportunity arises whenever the forward rate is less than 0.9100.

Alternatively, suppose the one year AUD/USD forward rate is more than 0.9100, say 0.9200. An arbitrage profit can be achieved by the following transactions:

Time	Description	Cash Flows	
0	Borrow USD 955,000 at 3.20%	USD	0
	Convert to AUD 1M and invest at 8.30%	AUD	0
	Enter into a 1-year forward contract to		
	buy USD 985,560 and		
	sell AUD 1,071,261 at 0.9200		
	INITIAL OUTLAY	AUD	0
1 year	Repay USD loan	USD	-985,560
	AUD investment matures	AUD	+1,083,000
	Receive USD from forward contract	USD	+985,560
	Outlay AUD from forward contract	AUD	-1,071,261
	INCOME	AUD	+ 11,739

Again, an arbitrage profit can be achieved from a zero dollar outlay whenever the forward rate is greater than the rate determined from the interest rate differential.

Reconstructing the forward rate

When the AUD/USD forward rate is equal to 0.9100, the rate implied by the interest rate differential, no arbitrage opportunity exists. To confirm this, a zero profit is realised through the following transactions:

Time	Description	Cash Flows	
0	Borrow AUD 1M at 8.30%	AUD	0
	Convert to USD 955,000 and invest at 3.20%	USD	0
	Enter into a 1-year forward contract to		
	buy AUD 1,083,000 at 0.9100 and		
	sell USD 985,530		
	INITIAL OUTLAY	USD	0
1 year	Repay AUD loan	AUD	-1,083,000
	USD investment matures	USD	+985,560
	Receive AUD from forward contract	AUD	+1,083,000
	Outlay USD from forward contract	USD	-985,530
	INCOME	USD	+ 30

In this case, the profit should be zero. The small profit observed above is due to rounding.

Similarly, a portfolio of the reverse transactions could be constructed as follows:

Time	Description	Cash Flows	
0	Borrow USD 955,000 at 3.20%	USD	0
	Convert to AUD 1M and invest at 8.30%	AUD	0
	Enter into a 1-year forward contract to buy USD 985,560 and sell AUD 1,083,033 at 0.9100		
	INITIAL OUTLAY	AUD	0
1 year	Repay USD loan	USD	-985,560
	AUD investment matures	AUD	+1,083,000
	Receive USD from forward contract	USD	+985,560
	Outlay AUD from forward contract	AUD	-1,083,033
	INCOME	AUD	- 33

Again, a zero profit (other than rounding) is observed where the forward rate equals the rate implied by the interest rate differential.

Given the spot foreign exchange rate and the interest rate for each country, a forward foreign exchange rate can be calculated. The only deviation, from this rate, that should exist, will be the margin (buy/sell spread) applied by the financial intermediary. Any additional difference would give rise to arbitrage opportunities and thus risk-free profit by constructing similar portfolios to those used in the examples above.

Calculating the Forward Rate

Mathematically the formula for determining the forward rate is:

$$F_0 = \frac{S_0(1+r)^T}{(1+r_f)^T}$$

where: F_0 = today's forward foreign exchange rate
S_0 = today's spot foreign exchange rate
r = the domestic interest rate (dependant on the method of quotation)
r_f = the foreign interest rate (dependant on the method of quotation)
T = the time period over which the interest rates apply

A Note About FX Complexity

An added complexity is the fact that the way Australia quotes its exchange rates, is from the perspective of the foreign currency. Hence, in our example above, 0.9550 is quoted as USD per AUD or if you were an American, how much does it cost to buy one Australian dollar. Hence, if we are using rates quoted from the foreign country's perspective, then the domestic interest rate will be the US interest rate and the foreign interest rate will be the Australian interest rate.

Continuing our example, the forward rate is calculated as:

$$F_0 = \frac{0.9550(1+0.032)^1}{(1+0.083)^1}$$

That is to say, take 95.5 US cents, multiply by the US interest rate (to get our USD return) then divide by our funding cost of 8.3% (the Australian interest rate).

$$F_0 = 0.9100$$

This rate should be familiar, from the no-arbitrage example above.

An extension to the discrete compounding interest calculation above, is to use continuous compounding rates as is used in financial markets, hence the formula is:

$$F_0 = S_0 e^{(r - r_f)T}$$

where:
- F_0 = today's forward foreign exchange rate
- S_0 = today's spot foreign exchange rate
- r = the continuously compounded domestic interest rate (dependent on the method of quotation). This is equal to ln(1 + 0.032) or 0.31499
- r_f = the continuously compounded foreign interest rate (dependent on the method of quotation). This is equal to ln(1 + 0.083) or 0.079735
- T = the time period over which the interest rates apply

Here the current exchange rate is affected by the funding and earning interest rates, as before, but at continuous interest rates. So the current exchange rate grows at a continuously compounded rate over a given period of time.

Continuing our example, the forward rate is calculated as:

$$F_0 = 0.9550 e^{(0.031499 - 0.079735) \times 1}$$

$$F_0 = 0.9550 e^{-0.048236}$$

← A negative value decays or reduces the current exchange rate.

$$F_0 = 0.9100$$

When using continuous rates we can see more clearly how the interest rate differential affects the forward rate. Multiplying a number by an exponential raised to a **positive** number will create a **growth** rate, but raised to a **negative** number creates a **decay** rate. Therefore, when Australian interest rates are above US interest rates the forward exchange rate will be lower than the current exchange rate and vice versa.